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TECHNICAL REPORT

WSRL-0182-TR

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A.K. STEELE

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OPTIMUM BEAMFORMING SUBJECT TO MULTIPLE LINEAR CONSTRAINTS

A.K. Steele

SUMMARY

Optimum beamformers with a single look direction constraint can suffer from signal suppression problems when the optimum weights are claculated from the inverse of the signal-plus-noise cross-spectral matrix. Signal suppression occurs when the beam steer direction does not exactly correspond to the signal direction and this can occur if the number of fixed beams is small. The use of multiple linear constraints upon the optimum weights reduces this signal suppression. Multiple directional constraints can lead to ill-conditioned equations. However it is shown that the limiting solutions of multiple directional constraints are multiple derivative constraints and these do not lead to ill-conditioned equations. The ability of various derivative constraints to prevent signal suppression is analysed quantitatively.

POSTAL ADDRESS: Chief Superintendent, Weapons Systems Research Laboratory, Box 2151, GPO, Adelaide, South Australia, 5001

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Optimum beamformers with a single look direction constraint can suffer from signal suppression problems when the optimum weights are calculated from the inverse of the signal-plus-noise cross-spectral matrix. Signal suppression occurs when the beam steer direction does not exactly correspond to the signal direction and this can occur if the number of fixed beams is small. The use of multiple linear constraints upon the optimum weights reduces this signal suppression. Multiple directional constraints can lead to ill-conditioned equations. However it is shown that the limiting solutions of multiple directional constraints are multiple derivative constraints and these do not lead to ill-conditioned equations. The ability of various derivative constraints to prevent signal suppression is analysed quantitatively.

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1. INTRODUCTION

Optimum beamforming with multiple linear constraints is now a well-established technique in array processing. In the simplest case, a single constraint is imposed, viz. unity signal power response in the beam steer direction; the weight vectors are then calculated by minimising the beamformer output power subject to this constraint. This first-generation optimum beamformer is also optimum in the sense that it (a) maximises the output signal-to-noise ratio (SNR) and (b) gives a least-mean-square (LMS) estimate of the signal(ref.1). These beamformers are usually designed under the assumptions of plane-wave signals and an ideal propagation medium. In actual operating conditions these ideal assumptions do not hold, and signal suppression can arise from causes such as beam steer angle errors, phase errors in the beamformer, and multipath propagation.

When there are only a finite number of beams to span the total bearing angle, any signal that is not exactly aligned with one of the beam steer directions will be regarded as an unwanted interference by the beamformer and therefore will tend to be suppressed. To overcome this problem, it is desirable to broaden the signal acceptance angle (ie the width of the main beam) whilst preserving the beamformer's ability to reject interference from directions outside this acceptance angle. There are two ways of achieving this, viz.(i) imposition of a magnitude constraint on the weight vector norm, and (ii) imposition of multiple linear constraints so as to preserve the main beam shape. The former method also improves the robustness of the beamformer to phase errors, whereas the latter method does not. This report is only concerned with the prevention of signal suppression arising from beam steer angle errors by the use of multiple linear constraints, and in particular, investigates the choice of these constraints.

The work reported is part of a continuing programme of research of techniques of signal processing for underwater detection carried out under Task No. DST 79/069.

2. OVERVIEW

2.1 Notation

Denote by $x = \{x_j\}$ the column vector representing the output of the K sensors at some frequency f (and corresponding wavelength λ). The cross-spectral matrix R of the sensor outputs is defined by*

$$R = \langle x x^H \rangle \quad . \tag{1}$$

Denote by z the K-vector of beamformer weights. The average power out of the beamformer is then given by

$$P = \langle z^H x x^H z \rangle = z^H \langle x x^H \rangle z = z^H Rz \quad . \tag{2}$$

For a unit magnitude plane-wave signal incident upon the array, the cross-spectral matrix R is

$$R = u u^H \quad , \tag{3}$$

*H denotes the conjugate transpose of either a vector or a matrix.

where u is the signal vector at the K sensors;

$$u_j = \exp(i2\pi f \tau_j) , \tag{4}$$

τ_j being the time of arrival (relative to an arbitrary reference point) of the signal at the j-th sensor. In general

$$R = Q + \sigma^2 u u^H \tag{5}$$

where Q is the normalised noise cross-spectral matrix and σ^2 is the signal-to-noise ratio.

2.2 Beamformers

For an unshaded conventional beamformer, the weight vector z_c takes the form

$$z_c = \frac{1}{K} v \tag{6}$$

where v is the so-called steering vector;

$$v = \exp [i\xi_j(\theta,\phi)] \tag{7}$$

where $\xi_j(\theta,\phi)$ is the phase of a plane-wave signal received from the steered direction (θ,ϕ) by the j-th sensor. If the conventional beamformer is steered in the direction of a signal, the signal power p_s out of the beamformer is

$$p_s = z^H u u^H z = \frac{1}{K^2}|u^H u|^2 = 1 , \tag{8}$$

and so the conventional beamformer has unity response in the look direction.

The single constraint optimum beamformer uses a weight vector z_o given by (ref.1,2)

$$z_o = Q^{-1}v/v^H Q^{-1}v , \tag{9}$$

where Q is the noise cross-spectral matrix. It is easily shown that this optimum beamformer has unity response in the look direction v. A unity look direction response is actually a single linear constraint on the weight vector z of the form

$$v^H z = 1 \tag{10}$$

Equation (9) can be derived by either

(i) minimising the noise power $z^H Qz$ subject to the linear constraint (10), or

(ii) maximising the array gain subject to the constraint (10), or

(iii) finding the weight vector that gives an LMS estimate of a unit strength signal in the look direction.

All three methods give the same solution(ref.1,2) and if the steering vector v and signal model vector u are the same, then the array gain g_o of the optimum beamformer is

$$g_o = u^H Q^{-1} u . \tag{11}$$

3. *THE SIGNAL SUPPRESSION PROBLEM*

3.1 Theoretical Development

In a number of practical situations, the cross-spectral matrix Q of the noise alone cannot be estimated since the signal components cannot be extracted from the observed cross-spectral matrix R. Thus the optimum weight vectors must be calculated from the inverse of the signal-plus-noise cross-spectral matrix. Furthermore, any plane-wave u not incident from the chosen steer direction v is treated as an interference by the optimum beamformer and will tend to be *rejected*. Thus if the chosen steering direction does not exactly match the signal direction then signal suppression occurs. This effect has been investigated by Cox(ref.3) who derived the results of this section.

For any weight vector z and a signal model vector u, the array gain is

$$g = \frac{|z^H u|^2}{z^H Qz} \tag{12}$$

Consider first the ideal case where the cross-spectral matrix Q of the noise alone can be estimated. For a given steering direction (not necessarily the signal direction) with associated steering vector v, the optimum weight vector z_o is given by equation (9). Substitution of equation (9) into equation (12) gives the gain of the single constraint optimum beamformer as

$$g_Q = g_u \left(\frac{|g_{uv}|^2}{g_u \, g_v} \right) , \tag{13}$$

where

$$g_u = u^H Q^{-1} u, \quad g_v = v^H Q^{-1} v, \tag{14(a)}$$

and

$$g_{uv} = g_{vu}^{*} = u^H Q^{-1} v \quad . \tag{14(b)}$$

The notation g_Q is used here to indicate that the weight vector is calculated from the inverse of the noise only cross-spectral matrix. The first term g_u in equation (13) is the gain when u and v are aligned (ie the beam is steered in the direction of the signal), and is the same as that given by equation (11). The second term in equation (13) expresses the loss in gain due to the mismatch be between u and v and is independent of the signal strength; this term is the polar response of the optimum beamformer.

Now as discussed above, let the weight vectors be calculated from the inverse of the signal-plus-noise cross-spectral matrix R which is given by

$$R = Q + \sigma^2 uu^H , \tag{15}$$

where Q is the normalised noise cross-spectral matrix and σ^2 is the signal-to-noise ratio. The weight vector z_o can be calculated from

$$z_o = R^{-1} v / v^H R^{-1} v \tag{16}$$

by the use of Woodbury's identity:

$$(Q + \alpha uu^H)^{-1} = Q^{-1} - \frac{\alpha Q^{-1} uu^H Q^{-1}}{1 + \alpha u^H Q^{-1} u} \tag{17}$$

Substitution of equation (16) into equation (12) gives the gain of the beam-former as

$$g_R = \frac{g_u \left(\dfrac{|g_{uv}|^2}{g_u g_v} \right)}{1 + (2\sigma^2 g_u + \sigma^4 g_u^2)\left(1 - \dfrac{|g_{uv}|^2}{g_u g_v}\right)} \tag{18}$$

where the notation g_R is used to indicate that the weight vector is calculated from the inverse of the signal-plus-noise cross-spectral matrix.

If the steering vector v is aligned with the signal model vector u, then

$$g_u = g_v = g_{uv}$$

and so $g_R = g_u$, ie there is no signal suppression even though the inverse of the signal-plus-noise cross-spectral matrix was used to calculate the weights.

Cox expresses the term $|g_{uv}|^2/g_u g_v$ as the cosine-squared of the generalised angle between u and v in a space where the inner product between two vectors a and b is defined by $a^H Q^{-1} b$. Although this gives a very elegant method of analysing these problems, it will not be used here since different forms of inner product arise in multiple constraint problems and hence would make the comparison of the performance of different beamformers very difficult.

Instead, the results will be given as a function of the angular separation between two directions.

3.2 Examples

The example used in this report is a horizontal planar array (HPA) and consists of 25 sensors arranged in 5 concentric rings, with a radial spacing of r between adjacent rings and between the innermost ring and the origin. On each ring the five sensors are spaced evenly in azimuth with one sensor in the 0^o reference azimuth direction.

Figures 1 and 2 illustrate the signal suppression for beamformers that use the inverse of the noise only cross-spectral matrix and the signal-plus-noise cross-spectral matrix respectively for weight computation. In both cases the ambient noise field is taken to be spherically isotropic noise with uncorrelated noise -30 dB re the isotropic noise. In figure 1, the second terms of equation (13), ie $|g_{uv}|^2/g_u g_v$ is plotted as a function of the azimuthal difference between u and v ($\Delta\theta$) at a frequency corresponding to $r/\lambda = 0.2$. It can be seen that little signal suppression occurs, with the 3 dB loss point occurring at $\Delta\theta = 14^o$. In figure 2 the SNR out of the beamformer is plotted as a function of $\Delta\theta$ for a range of input SNR of -15 dB to +6 dB. It is immediately obvious that signal suppression becomes more series as the input SNR increases, so much so that, for a given $\Delta\theta$ a low input SNR can lead to a larger output SNR than a high input SNR. The aim of using multiple linear constraints is to reduce the severity of this signal suppression at the lower values of $\Delta\theta$.

4. MULTIPLE LINEAR CONSTRAINTS

4.1 Optimum weights

A set of L linear constraints on the weight vector z can be expressed as

$$V^H z = c , \tag{19}$$

where V is a K x L matrix whose columns are the constraint vectors and c is the L-vector of constraint values. The optimum weights z_o are chosen so as to minimise the beamformer total output power $z^H Rz$ subject to the constraints (19) being satisfied. The use of Lagrangian multipliers to solve this problem is well known (see, eg ref.2) and will not be repeated here. The optimum weight vectors are given by

$$z_o = R^{-1} V(V^H R^{-1} V)^{-1} c , \tag{20}$$

and the beamformer output power is

$$p = c^H (V^H R^{-1} V)^{-1} c . \tag{21}$$

Now let R be the sum of signal and noise cross-spectral matrices as given by equation (15). Then the beamformer output power is

$$p = c^H(V^H(Q + \sigma^2 uu^H)^{-1} V)^{-1} c . \tag{22}$$

The repeated application of Woodbury's identity (17) to the above equation then gives the result:

$$p = c^H(V^H Q^{-1} V)^{-1} c + \frac{\sigma^2 | c^H(V^H Q^{-1} V)^{-1} V^H Q^{-1} u |^2}{1 + \sigma^2 u^H Q^{-1} u - \sigma^2 u^H Q^{-1} V(V^H Q^{-1} V)^{-1} V^H Q^{-1} u} \tag{23}$$

Furthermore, if the i-th constraint is the signal model vector u (ie the i-th column of V is u), then it is not difficult to show that

$$(V^H Q^{-1} V)^{-1} V^H Q^{-1} u = e_i \tag{24}$$

where e_i is an elementary vector, ie a vector with unity in the i-th entry and zeros elsewhere. If the i-th constraint value is unity, then substitution of equation (24) into equation (23) gives the result that

$$p = c^H(V^H Q^{-1} V)^{-1} c + \sigma^2 \tag{25}$$

and from this it can be seen that minimisation of the total output power also minimised the output noise power. Thus a beamformer derived from the latter minimisation is equivalent to one derived from the former minimisation if there is a unity constraint in the direction of the incident signal. If the unity constraint direction(s) does (do) not correspond to the signal direction, then signal suppression will occur in the beamformer derived from minimisation of the total output power in a manner similar to that discussed in Section 3. This topic will be discussed further in Section 5.

Gray(ref.2) has recently shown that, provided that same set of linear constraints is imposed, one of which is a unity constraint value in the signal direction, then the following three different optimisation criteria lead to the same solution for the optimum weight vector. They are:

(i) minimising the total output power,

(ii) a least mean square fit to the signal vector, and

(iii) maximisation of the output SNR.

Thus all the results in this report are applicable to beamformers designed upon any one of these three criteria.

4.2 Directional and derivative constraints

In order to make the optimum beamformer robust against beam steering angle errors, many authors (eg Vural, ref.4) have suggested the use of either multiple directional constraints or multiple derivative constraints upon the polar response of the beamformer. The aim of this is to attempt to increase the angle over which the beamformer accepts a directional source as a signal and does not reject it as an interference. This can be achieved to some degree by maintaining the main lobe of the polar response and not allowing a null to form until the difference between the steer direction and the signal direction becomes substantial.

A two-point unity directional constraint centred on the steer direction θ can be expressed by

$$V = [v(\theta - \Delta) : v(\theta + \Delta)], \quad c = (1\ 1)^H \tag{26}$$

where the v( ) are steering vectors.

Similarly, a three-point unity directional constraint centred on the signal direction θ can be expressed by

$$V = [v(\theta - \Delta) : v(\theta) : v(\theta + \Delta)], \tag{27}$$

$$c = (1\ 1\ 1)^H .$$

The beamformer performance is relatively insensitive to the magnitude of Δ for small values of Δ. (For the HPA used in this report very little difference in performance existed for values of Δ up to approximately 10^o for r/λ less that 0.4.) However, if Δ is too large, very strong signals in directions between the constraint directions may cause the beam to split into multiple beams with nulls in the signal directions. Thus only small values of Δ are of interest. However, when Δ is very small, the matrix $V^H R^{-1} V$ becomes ill-conditioned since the columns of V are approaching linear dependence. This problem can be overcome by noting that the limit $\Delta \to 0$ of a two-point directional constraint is a derivative constraint of the form

$$\tilde{V} = [v(\theta) : \dot{v}(\theta)], \quad \tilde{c} = (1\ 0)^H , \tag{28}$$

where $\dot{(\ )}$ denotes differentiation with respect to θ. Similarly, the limit $\Delta \to 0$ of a three-point directional constraint is the multiple derivative constraint:

$$\tilde{V} = [v(\theta) : \dot{v}(\theta) : \ddot{v}(\theta)], \quad \tilde{c} = (1\ 0\ 0)^H . \tag{29}$$

(The proof for the limit of a three-point directional constraint is set out in Appendix I. The proof for a two-point constraint is similar.) Since the vectors $v(\theta)$, $\dot{v}(\theta)$ and $\ddot{v}(\theta)$ are linearly independent, the matrix $\tilde{V}^H R^{-1} \tilde{V}$ will not be ill-conditioned.

Another form of multiple directional constraint was discussed by Vural(ref.5). In this case the constraint values are those equal to the response (magnitude and phase) of a conventional beamformer, ie for three directional constraints centred on the signal direction θ, the matrices V and c are

$$V = \left[v(\theta - \Delta) \ : \ v(\theta) \ : \ v(\theta + \Delta) \right] ,$$

and

$$c = \begin{bmatrix} v(\theta - \Delta)^H \ v(\theta)/K \\ 1 \\ v(\theta + \Delta)^H \ v(\theta)/K \end{bmatrix} . \tag{30}$$

The limiting solution for $\Delta \to 0$ for this form of directional constraint is the multiple derivative solution (see Appendix I):

$$\tilde{V} = \left[v(\theta) \ : \ \dot{v}(\theta) \ : \ \ddot{v}(\theta) \right] ,$$

$$\tilde{c} = \begin{bmatrix} 1 \\ \dot{v}(\theta)^H \ v(\theta)/K \\ \ddot{v}(\theta)^H \ v(\theta)/K \end{bmatrix} . \tag{31}$$

The term $\dot{v}^H \ v/K$ is purely imaginary and its magnitude increases linearly with r/λ . (This term must have a zero real component for the polar power response to have a maximum in the direction θ.) The term $\ddot{v}^H v/K$ has a dominant real component who magnitude increases as the square of r/λ .

Thus since the limiting solutions of multiple directional constraints are derivative constraints, and the fact that the latter form of constraint does not lead to ill-conditioned equations, only derivative constraints will be discussed further.

4.3 Discussion and examples

An insight to the effect of the various derivative constraints mentioned above can be obtained by examining the behaviour of these beamformers in a highly directional noise field. For the following examples (figures 3 to 11), the noise model used is spherically isotropic noise plus uncorrelated noise -30 dB re the isotropic noise plus a directional source 10 dB above the total noise power. The azimuth of the source is 180° and $r/\lambda = 0.2$. All the beamformers have one unity look direction constraint, so the numerator of equation (12) becomes unity. Substitution of equation (20) (with R=Q) into equation (12) then gives the maximum array gain as

$$g_o' = 1/c^H(V^H Q^{-1} V)^{-1} c . \tag{32}$$

Note that this is the reciprocal of the beamformer noise power output which is given by the first term of equation (25).

Figure 3 shows array gain vs asimuth curves for various constraints. They are:

(i) single look direction constraint (Type 1),

(ii) look direction constraint plus zero first derivative constraint (Type 2),

(iii) look direction constraint plus zero first and second derivative constraints (Type 3), and

(iv) look direction constraint plus constraints on the first and second derivatives equal to the conventional beamformer response (Type 4).

The increase in the null width of Types 2, 3 and 4 beamformers over Type 1 beamformer is quite marked, especially for Types 3 and 4. Also of note is the loss in gain in directions away from the source for the Type 3 beamformer (see explanation below). The effect of the various types of constraints can be easily seen in the beamformer polar response plots shown in Figures 4 to 9.

Figure 4 shows the polar diagram of a Type 1 beamformer with a single unity look direction constraint at 190° azimuth. The beamformer has a very deep null in the direction of the interfering source at 180° azimuth.

Figure 5 shows the effect of an extra zero first derivative constraint (Type 2). The null at 180° is not as deep and a large sidelobe has appeared at 160°.

Figure 6 shows the effect of zero first and second derivatives (Type 3). The polar response at 190° is now a point of inflection, and two large sidelobes have appeared. The null depth at 180° is now only about 4 dB. Figure 7 shows the polar plot for the same beamformer with a steer azimuth of 300°. The very broad main beam will lead to a loss in array gain against isotropic noise, and this is the cause of the gain loss noted above.

Figures 8 and 9 show the polar plots of a Type 4 beamformer for steering azimuths of 190° and 300° respectively. The response of a Type 4 beamformer at azimuths near interfering sources is similar to that of a Type 3, but at azimuths away from interfering sources, the Type 4 has a much smaller beamwidth (at this frequency anyway) and so the loss in gain against isotropic noise will be much smaller.

This loss in gain against isotropic noise can be seen more clearly in Figure 10 which shows, for a beam steered in the signal direction, the array gain in isotropic noise (with -30 dB of uncorrelated noise) vs r/λ for Types 1, 2, 3 and 4 beamformers.

There is no measurable difference in gain between Type 1 and Type 2 beamformers. The lower curve shows the loss in gain due to the very large beamwidth of the Type 3 beamformer. At high frequencies, the Type 4 gain approaches that of a Type 1 and 2, whilst at low frequencies its gain is similar to that of a Type 3. This is to be expected since the beamwidth of a conventional beamformer increases as r/λ decreases and hence the beamwidth of the Type 4 beamformer will increase with decreasing r/λ. An alternative explanation is that the constraint values $\dot{v}^H v/K$ and $\ddot{v}^H v/K$ in equation (31) approach zero as r/λ decreases and hence the performance of a Type 4 beamformer will approach that of a Type 3 as r/λ approaches zero.

5. SIGNAL SUPPRESSION IN CONSTRAINED BEAMFORMERS

5.1 Theoretical development

The analysis of signal suppression which was done in Section 3 can easily be extended to multiple constraint beamformers.

For a noise-only cross-spectral matrix Q, the optimum weights are given by equation (20) with R=Q. Substitution of this weight vector equation into equation (12) gives

$$g'_Q = \frac{|c^H(V^H Q^{-1} V)^{-1} V^H Q^{-1} u|^2}{c^H(V^H Q^{-1} V)^{-1} c} \tag{33}$$

If a constraint is the signal model vector u and the corresponding constraint value is 1, then from equation (24) the numerator of equation (33) is unity and so equation (33) reduces to equation (32) as expected.

Let the signal-plus-noise cross-spectral matrix R be given by equation (15). Then substitution of equation (15) and equation (20) into equation (12) and repeated use of Woodbury's identity gives:

$$g'_R = \frac{|c^H(V^H Q^{-1} V)^{-1} V^H Q^{-1} u|^2}{(1+\sigma^2\gamma)^2 c^H(V^H Q^{-1} V)^{-1} c + \sigma^4\gamma |c^H(V^H Q^{-1} V)^{-1} V^H Q^{-1} u|^2} \tag{34}$$

where

$$\gamma = u^H Q^{-1} u - u^H Q^{-1} V(V^H Q^{-1} V)^{-1} V^H Q^{-1} u \ . \tag{35}$$

Equation (34) can be written in a more compact form, viz.,

$$g'_R = \frac{g'_Q}{(1+\sigma^2\gamma)^2 + \sigma^4\gamma\, g'_Q} \ . \tag{36}$$

Again, if a constraint is the signal model vector u and the constraint value is 1, then γ=0 and equation (36) reduces to equation (32).

If the constraint matrix reduces to the single constraint of Section 3 (ie V=v) then

$$g'_Q = \frac{|g_{uv}|^2}{g_v} = g_Q \ . \tag{37}$$

Also

$$\gamma = g_u - \frac{|g_{uv}|^2}{g_v}$$

$$= g_u - g_Q \, . \tag{38}$$

Hence from equation (36),

$$g'_R = \frac{g_Q}{[1 + \sigma^2 (g_u - g_Q)]^2 + \sigma^4 g_Q (g_u - g_Q)}$$

$$= \frac{g_Q}{1 + (2\sigma^2 + \sigma^4 g_u)(g_u - g_Q)} \quad , \tag{39}$$

which is just equation (18).

5.2 Discussion and examples

In the following examples, the noise field is spherically isotropic noise plus uncorrelated noise -30 dB re the isotropic noise and $r/\lambda = 0.2$.

Figure 11 illustrates the signal suppression for Types 1, 2, 3 and 4 beamformers whose weights are calculated from the noise-only cross-spectral matrix. The expression in equation (33) is plotted as a function of $\Delta\theta$ and has been normalised (ie g'_Q/g'_0 is plotted). The plots illustrate the increase in beamwidth of a Type 3 over a Type 4 and of a Type 4 over Types 1 and 2. The difference in beamwidth between Types 1 and 2 is not significant.

For the signal-plus-noise case the use of equation (34) provides a method of quantitatively comparing the use of derivative constraints to prevent signal suppression. Figures 12, 13 and 14 show plots of output SNR from Types 2, 3 and 4 beamformers respectively for a range of input SNR of -15 dB to +6 dB. These figures should be compared with figure 2 which shows the results for a Type 1 beamformer. It can be seen that all three forms of derivative constraint prevent the very sharp decrease in output SNR that occurs in a Type 1 beamformer for large values of input SNR and small values of $\Delta\theta$. The prevention of signal suppression in these regions is a highly desirable property of any type of beamformer. For $\Delta\theta$ equal to zero, the Type 3 beamformer shows a loss in output SNR compared with the Type 2 beamformer and this was noted in the previous section; however the Type 3 does prevent signal suppression for much larger values of $\Delta\theta$. The differences between Types 3 and 4 are negligible except that the Type 4 does not suffer the gain loss of the Type 3 at this value of r/λ ; for this reason the use of a a Type 4 beamformer is preferable to the use of a Type 3.

6. CONCLUSIONS

It has been shown that the use of multiple linear constraints reduces signal suppression when using optimally weighted beams. Derivative constraints are preferable to directional constraints since the latter can lead to ill-conditioned solutions. The use of weight vector norm constraints, either alone or in combination with linear constraints, has yet to be investigated.

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APPENDIX I

LIMITING SOLUTION FOR DIRECTIONAL CONSTRAINTS

The steering vectors for three constraint directions can be approximated by the first three terms of a Taylor series, ie

$$
\begin{aligned}
v(\theta - \Delta) &= v(\theta) - \dot{v}(\theta) + \tfrac{1}{2}\Delta^2 \ddot{v}(\theta) , \\
v(\theta) &= v(\theta) , \qquad\qquad (I.1) \\
v(\theta + \Delta) &= v(\theta) + \dot{v}(\theta) + \tfrac{1}{2}\Delta^2 \ddot{v}(\theta) .
\end{aligned}
$$

These equations can be rewritten as the matrix equation:

$$V = \tilde{V} D \qquad (I.2)$$

where

$$D = \begin{bmatrix} 1 & 1 & 1 \\ -\Delta & 0 & \Delta \\ \dfrac{\Delta^2}{2} & 0 & \dfrac{\Delta^2}{2} \end{bmatrix} , \quad \tilde{V} = [\, v(\theta) : \dot{v}\;(\theta) : \ddot{v}(\theta)] \; . \qquad (I.3)$$

The optimum weight vectors are given by

$$z_o = R^{-1} V (V^H R^{-1} V)^{-1} \; c \; . \qquad (I.4)$$

Substitution of equation (I.2) into equation (I.4) gives

$$
\begin{aligned}
z_o &= R^{-1} \; \tilde{V} D (D^H \tilde{V}^H R^{-1} \tilde{V} D)^{-1} \; c \\
&= R^{-1} \; \tilde{V} (\tilde{V}^H R^{-1} \tilde{V})^{-1} \; (D^{H-1} c) \; . \qquad (I.5)
\end{aligned}
$$

This latter equation is the equation for the weights of a derivative constrained beamformer with constraint values $\tilde{c}$ equal to

$$\tilde{c} = D^{H-1} c = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{1}{2\Delta} & 0 & \dfrac{1}{2\Delta} \\ \dfrac{1}{\Delta^2} & \dfrac{-2}{\Delta^2} & \dfrac{1}{\Delta^2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} . \qquad (I.6)$$

When the constraint values are those equal to the response of a conventional beamformer, c is given by

$$c = \begin{bmatrix} v(\theta - \Delta)^H v(\theta)/K \\ 1 \\ v(\theta + \Delta)^H v(\theta)/K \end{bmatrix} . \tag{I.7}$$

Substitution of equation (I.1) into this equation gives

$$c = \begin{bmatrix} (v^H v - \Delta \dot{v}^H v + \tfrac{1}{2} \Delta^2 \ddot{v}^H v)/K \\ 1 \\ (v^H v + \Delta \dot{v}^H v + \tfrac{1}{2} \Delta^2 \ddot{v}^H v)/K \end{bmatrix} , \tag{I.8}$$

and so

$$\tilde{c} = D^{H-1} c = \begin{bmatrix} 1 \\ \dot{v}^H v/K \\ \ddot{v}^H v/K \end{bmatrix} . \tag{I.9}$$

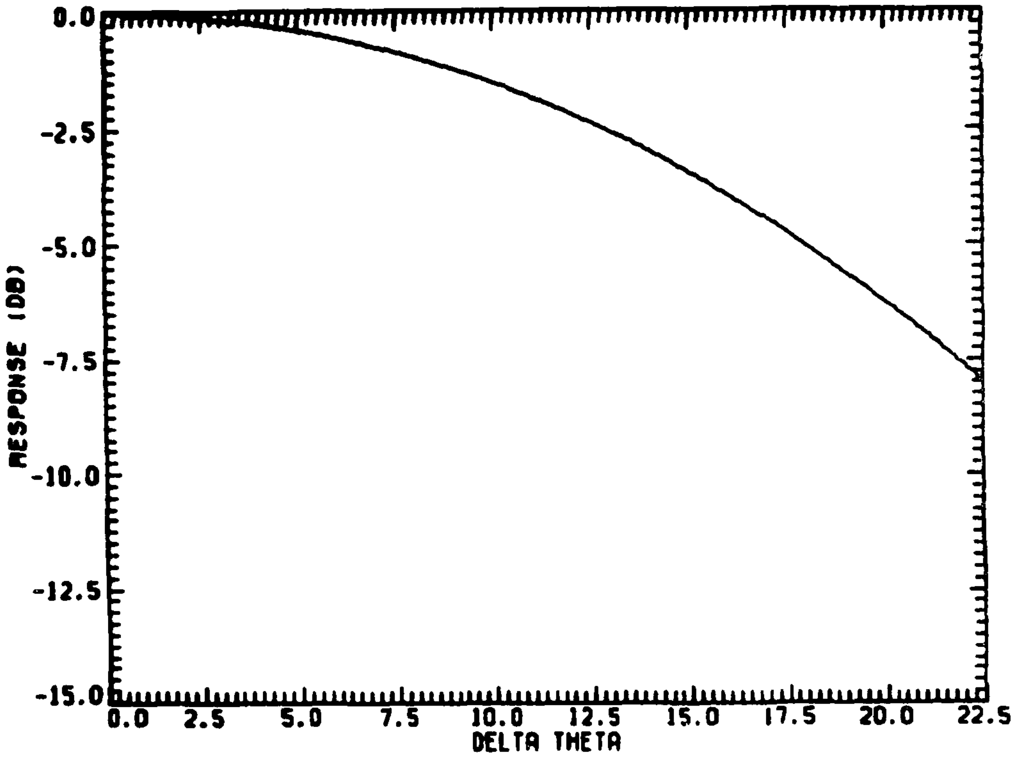

Figure 1. Signal suppression using weights calculated from noise cross-spectral matrix; single look direction constraint

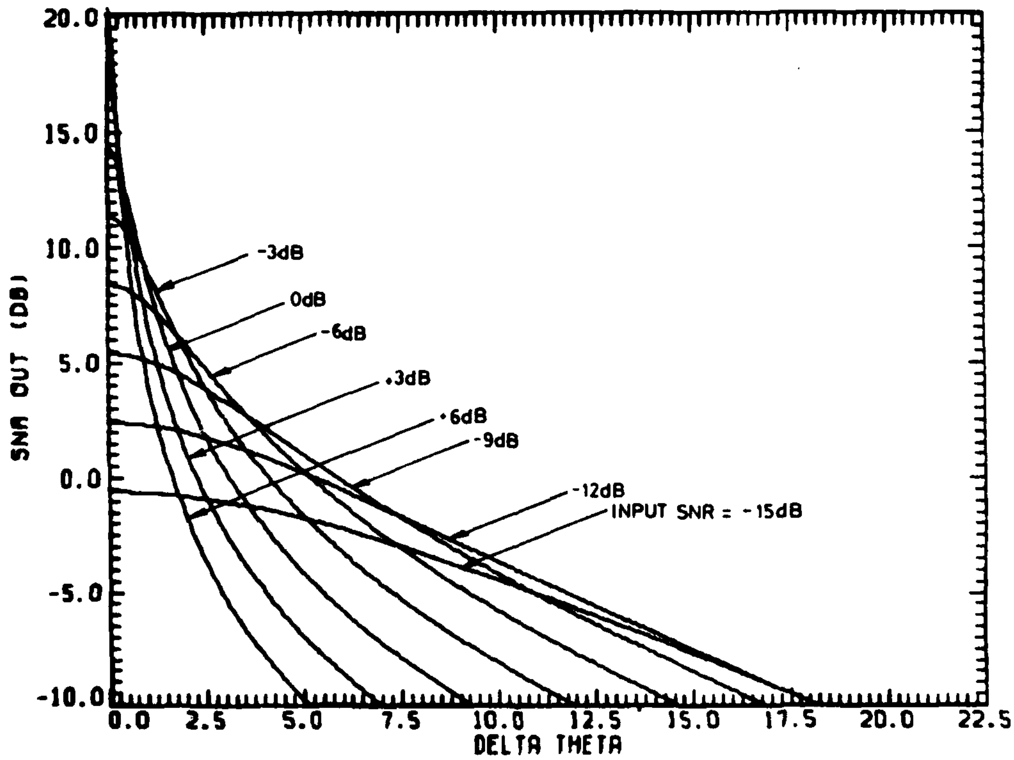

Figure 2. Signal suppression using weights calculated from signal-plus-noise cross-spectral matrix; single look direction constraint

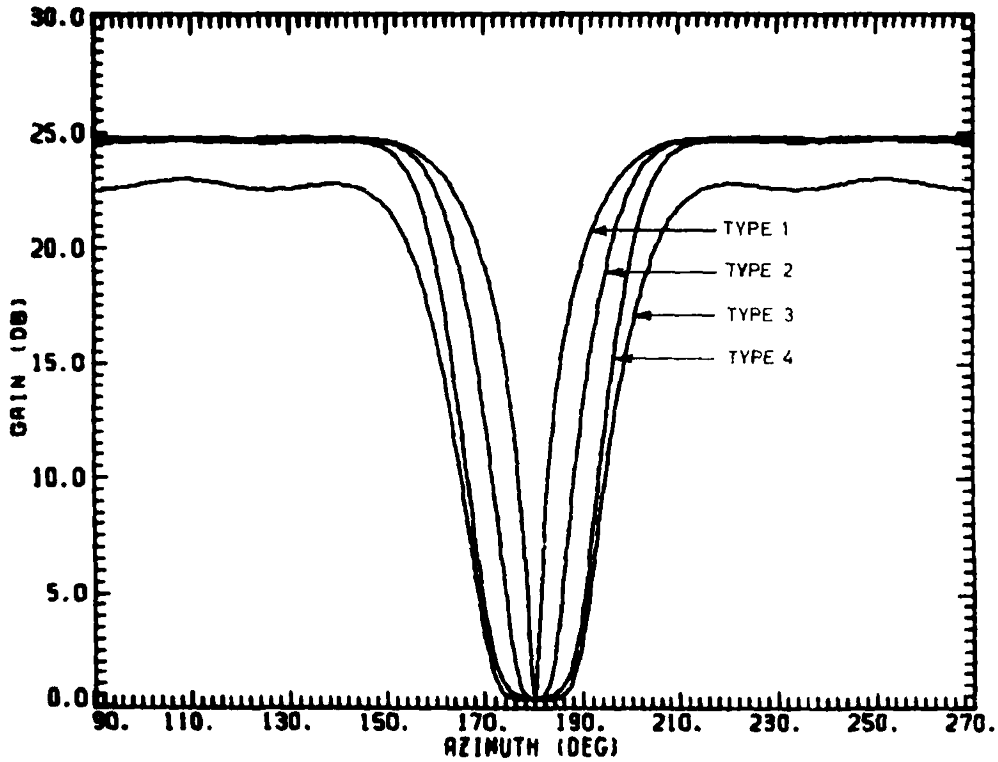

Figure 3. Array gain in a directional noise field; multiple constraint beamformers

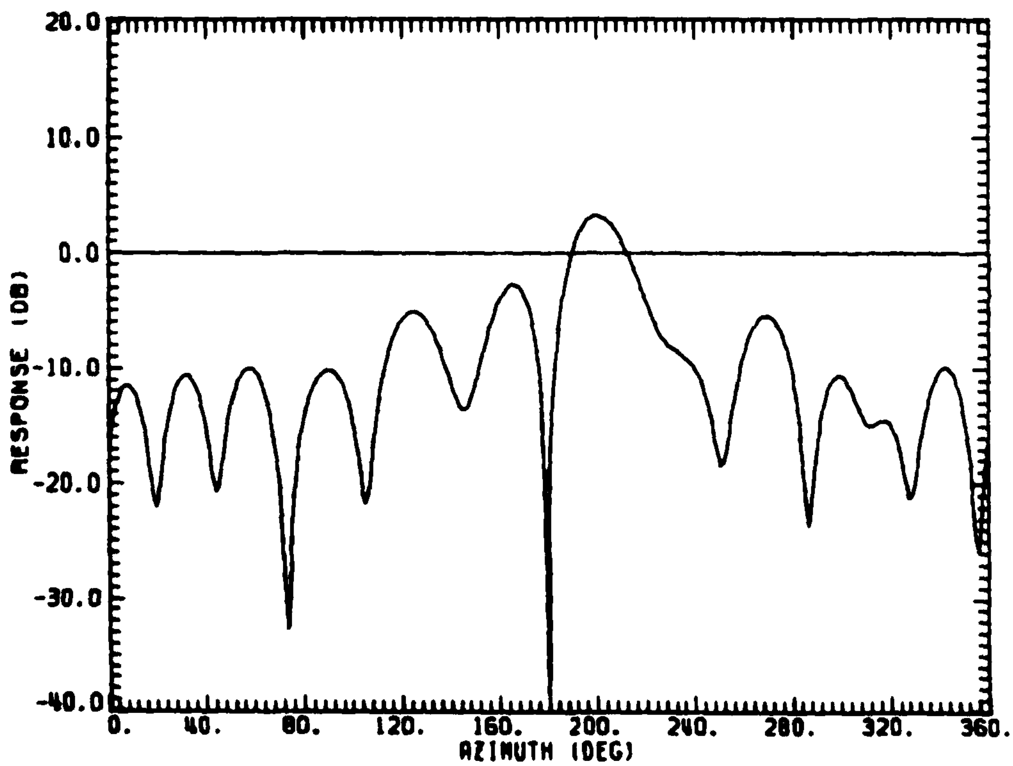

Figure 4. Polar diagram of a type 1 beamformer; steer azimuth = 190^{o}

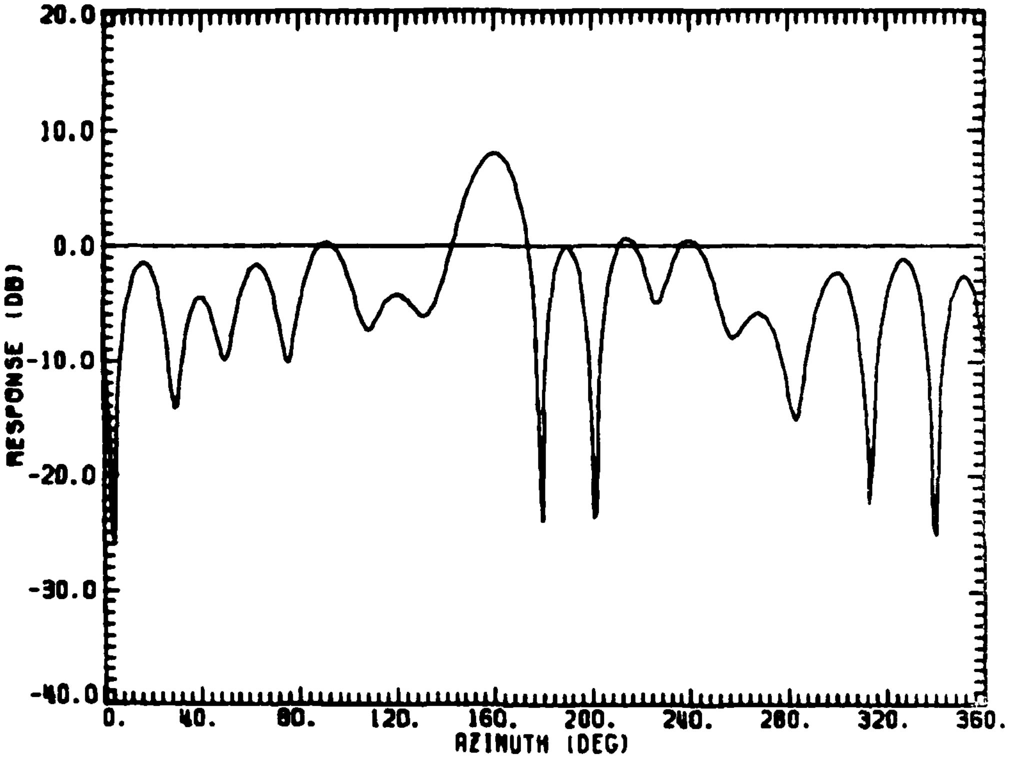

Figure 5. Polar diagram of a type 2 beamformer; steer azimuth = 190°

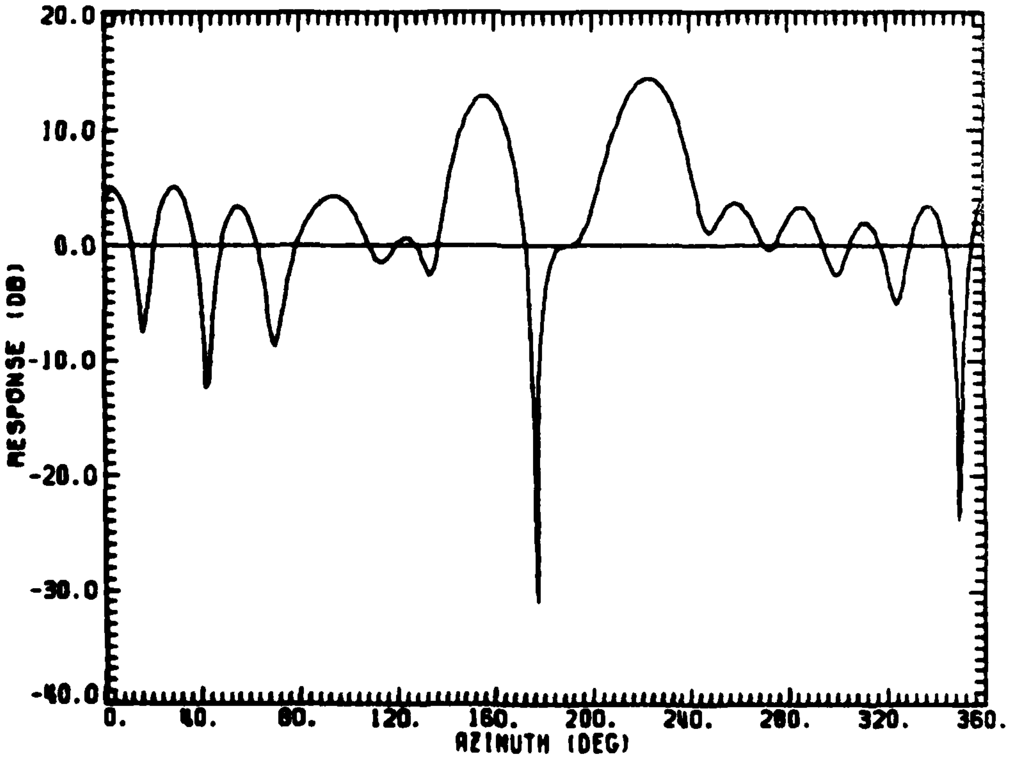

Figure 6. Polar diagram of a type 3 beamformer; steer azimuth = 190°

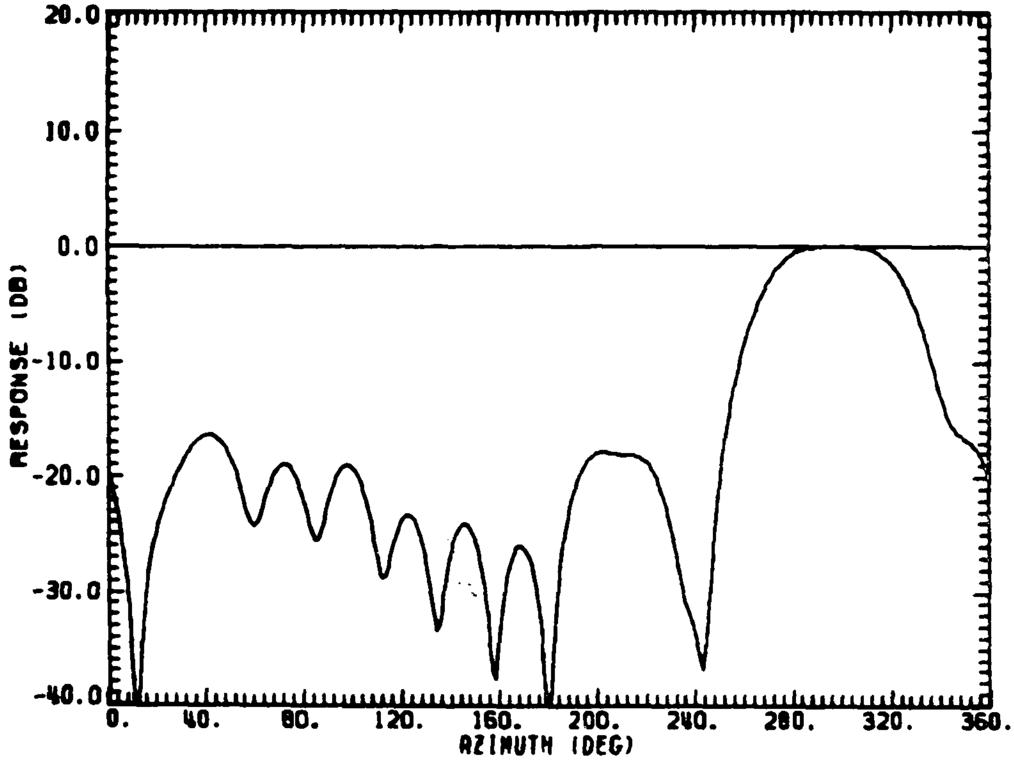

Figure 7. Polar diagram of a type 3 beamformer; steer azimuth = 300°

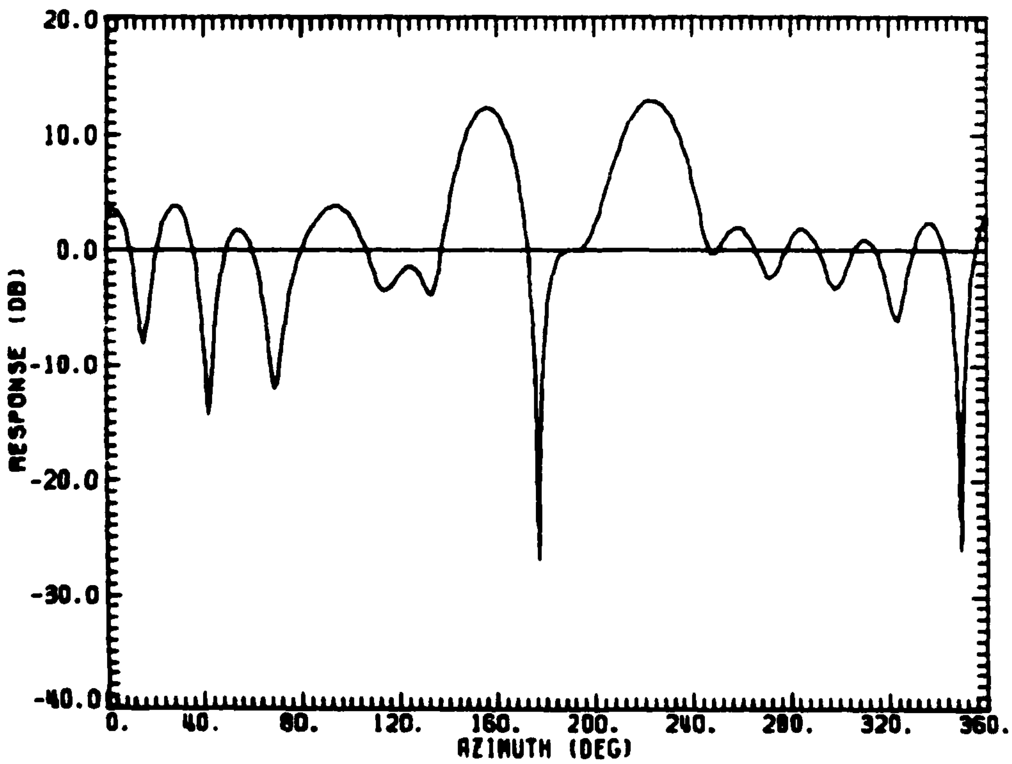

Figure 8. Polar diagram of a type 4 beamformer; steer azimuth = 190°

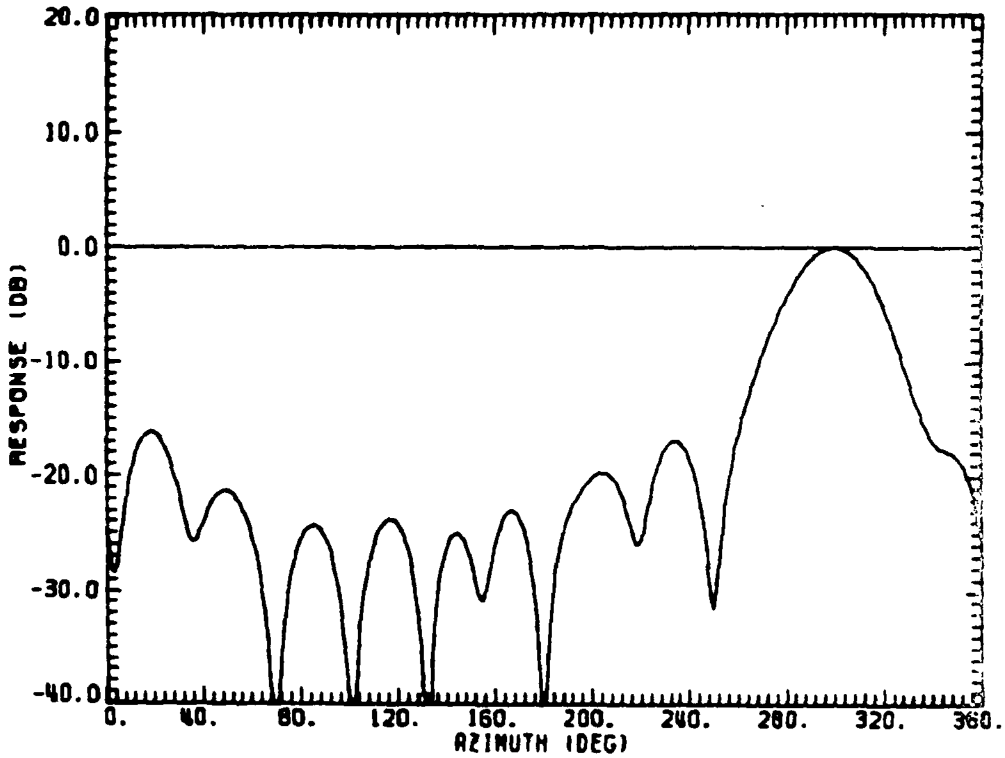

Figure 9. Polar diagram of a type 4 beamformer; steer azimuth = 300°

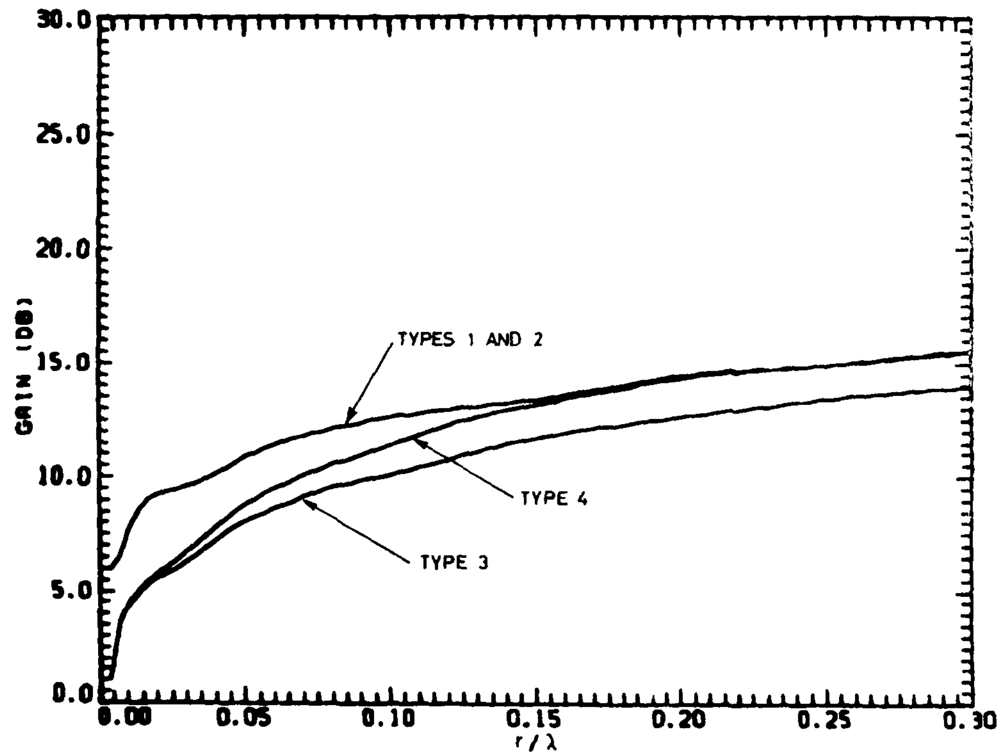

Figure 10. Array gain in isotropic noise; multiple constraint beamformers

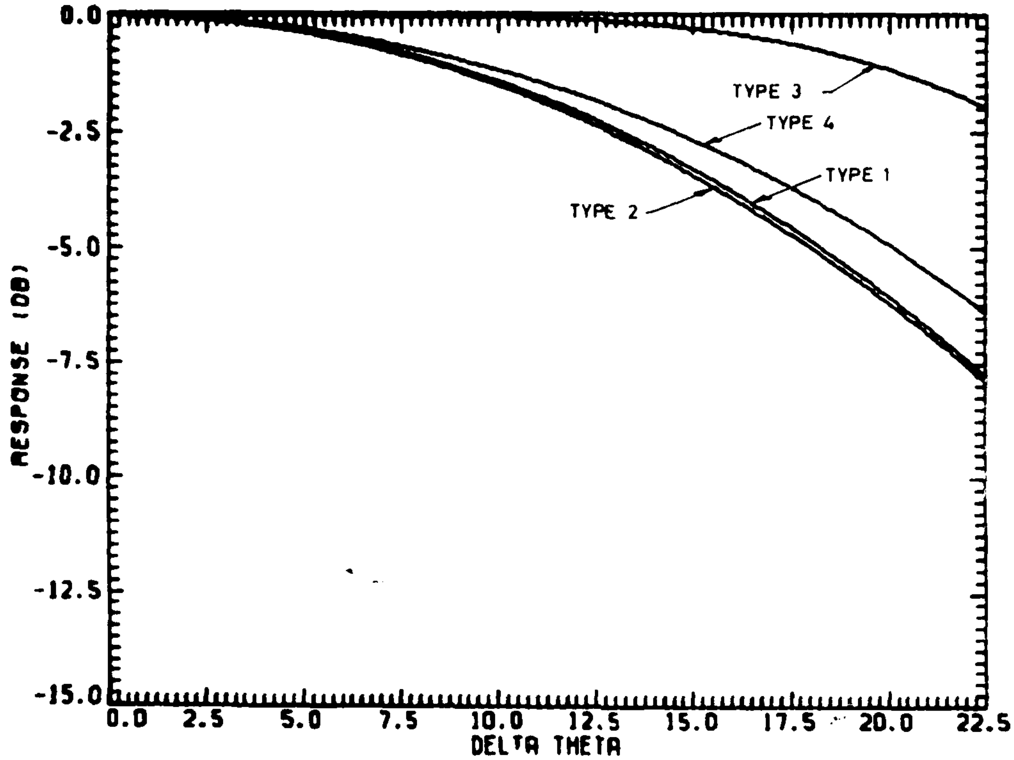

Figure 11. Signal suppression using weights calculated from noise cross-spectral matrix; multiple constraint beamformers

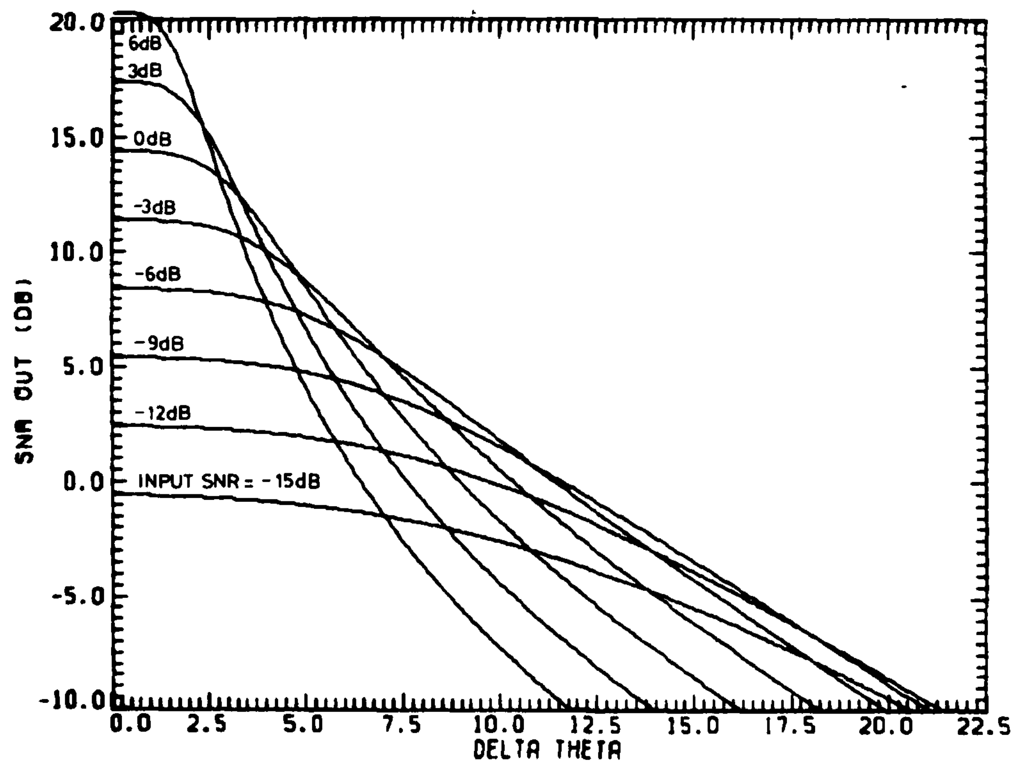

Figure 12. Signal suppression using weights calculated from signal-plus-noise cross-spectral matrix; type 2 beamformer

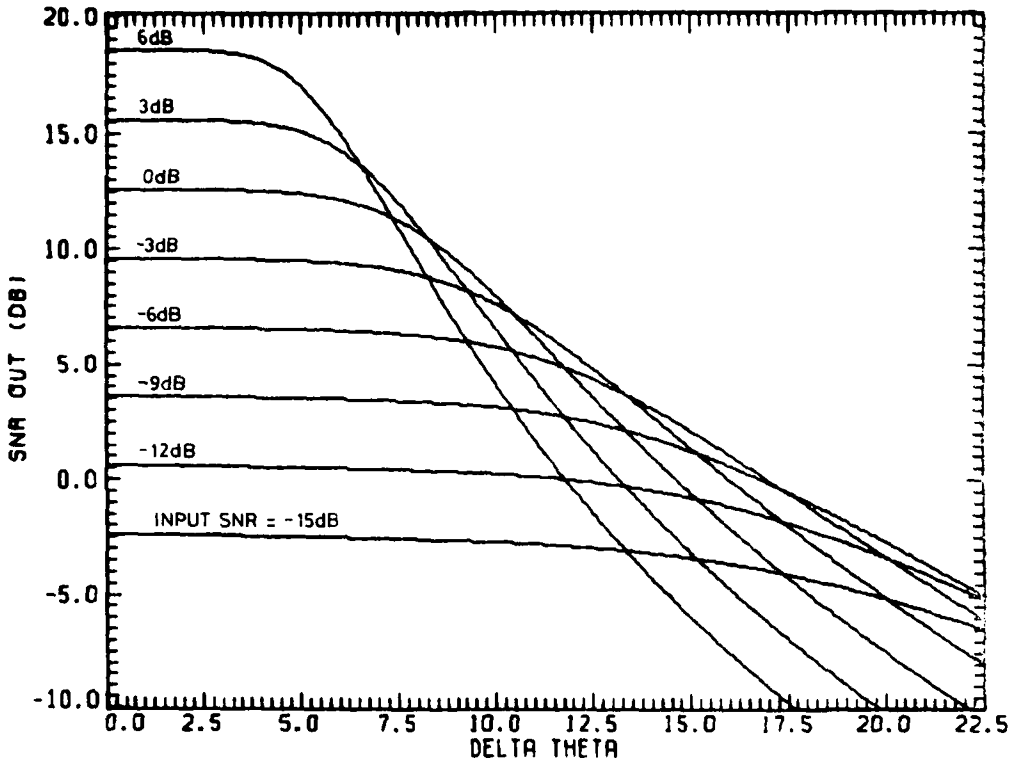

Figure 13. Signal suppression using weights calculated from signal-plus-noise cross-spectral matrix; type 3 beamformer

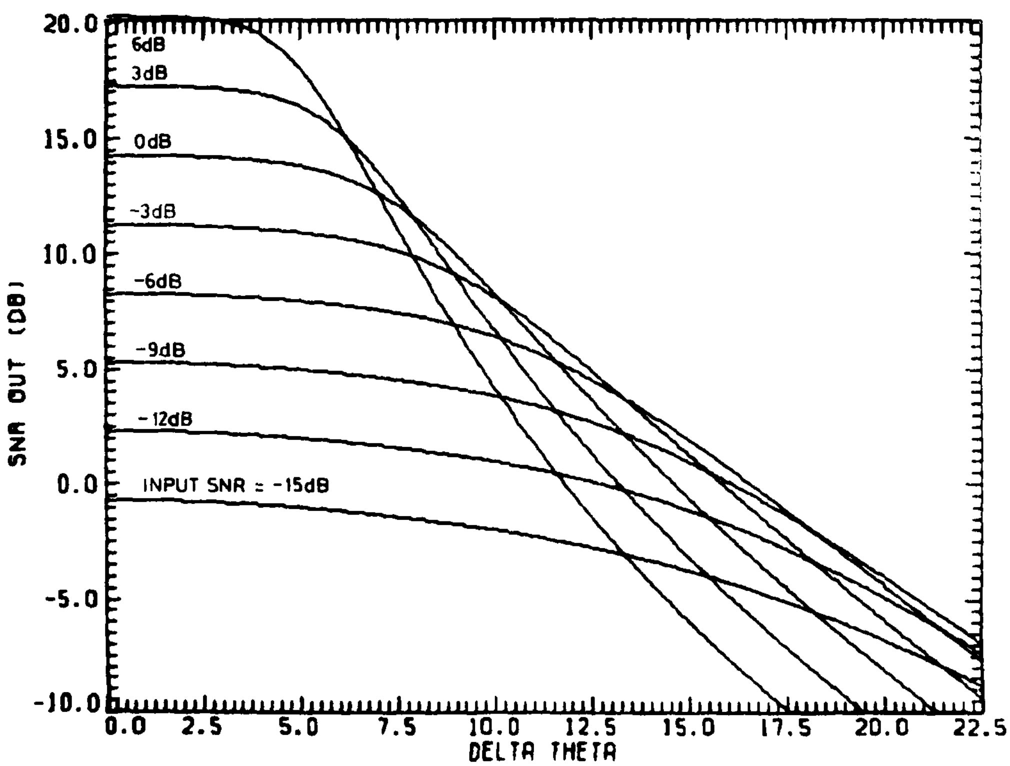

Figure 14. Signal suppression using weights calculated from signal-plus-noise cross-spectral matrix; type 4 beamformer

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